

**Invited Talk Abstract**

ICOML 2026 | July 27–29, 2026

**Pushing the Complexity Boundaries of Fixed-Point Equations: Adaptive Algorithm at the Tractability Frontier****Jelena Diakonikolas***University of Wisconsin-Madison*

<b>Date</b>	July 28, 2026
<b>Time</b>	10:00–10:30
<b>Session</b>	Session 4
<b>Venue</b>	S102, Lecture Hall, Gong-Guan Campus, NTNU

**Abstract**

High-dimensional settings of fixed-point operator equations  $T(x) = x$  arise across optimization, game theory, economics, and machine learning. When the operator  $T$  is contractive or nonexpansive, efficient algorithms with tight complexity guarantees are well established. For even mildly expansive operators, however, worst-case exponential lower bounds arise. This raises a natural question: where exactly does the tractability frontier lie? In this talk, I will present recent results that identify new tractable regimes beyond nonexpansiveness. I will introduce the Adaptive Gradual Halpern Algorithm (AdaGHAL), a fully parameter-free method that recovers optimal oracle complexities for contractive and nonexpansive operators while extending guarantees to mildly expansive operators up to the known hardness boundary. I will then introduce the class of gradually expansive operators—permitting Lipschitz constants up to approximately  $\sqrt{2}$ —and show that AdaGHAL finds  $\varepsilon$ -approximate fixed points in  $O(1/\varepsilon)$  iterations in this regime. These results hold in general normed spaces, including infinite-dimensional Banach spaces. I will conclude with implications for root-finding problems and weakly convex optimization, along with open questions.